(c) Prove that $\Gamma(n+1) = n\Gamma(n)$. Compute

(i) $\Gamma\left(-\frac{5}{2}\right)$

(ii) $\dfrac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$ (6+6+6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 5727 **K**

Unique Paper Code : 2512012301

Name of the Paper : Engineering Mathematics (DSC-7)

Name of the Course : **B.Sc. (H) Electronics (Core)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question **1** is compulsory.

3. Attempt any **5** questions including question **1**.

1. Attempt any **six**

(a) Solve

$$\frac{dy}{dx} = 2xy^2$$

for $y(0) = y_0$.

(b) Show that $\Gamma(n+1) = n!$

(c) Give the indical equation for

$$x^2\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} = 2y = 0$$

Also, evaluate 'r'.

(d) Verify the statement that, "The eigenvalues of A^{-1} of a square matrix has the inverse eigenvalues as A", using :

$$\begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix}$$

(b) Find the derivative of

$$f(z) = (3z^2 - 5) + i(4z + 1)$$

at $z = 2$

(i) using condition of differentiability

(ii) using differentiation rules

(c) Show that

$$v(x,y) = -\sin(x)\sinh(y)$$

is a harmonic function. Find the conjugate harmonic of v and the analytic function $f(z) = u + iv$.

(6+6+6)

4. (a) Check the exactness of the given differential equation and solve

$$(x - y)dx - dy = 0$$

for $y(0) = 2$.

(b) State Taylor's series Theorem and evaluate the Taylor Series for

$$f(z) = e^z$$

around $z = 0$.

5. (a) Determine the given function f(z) is analytic or not

$$f(z) + \frac{1}{2} ln(x^2 + y^2) + i tan^{-1} \left\{\frac{y}{x}\right\}$$

(b) Evaluate

$$\oint_C \frac{e^z}{(1 + z^2)}$$

around the following closed curves C

(i) |Z–i| = 1

(ii) |Z+i| = 1

(c) Show that

$$f(z) = |z^2|$$

is differentiable only at the origin. (6+6+6)

6. (a) Determine the solution for the given set of simultaneous linear equations using Gauss-Jordan Method

$$x + y - z = 3$$

$$3x + 2y - 2z = 8$$

$$2x - y - 3z = 7$$

(e) Use the Cauchy's n^{th} root test to determine the convergence of following series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots$$

(f) Use the integral method to determine the convergence of following series

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$$

(g) Show that

$$f(z) = \frac{1}{z}$$

is differentiable everywhere except at the origin. (3×6)

2. (a) Solve the ODE

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

(b) Determine

$(4x^3 + 6xy + y^2)dx + (3x^2 + 2xy + 2)dy = 0$

is exact and find its solution.

(c) Solve using series method the ODE

$$\frac{d^2y}{dx^2} + x^2y = 0$$

(6+6+6)

3. (a) Verify the Cayley-Hamiltonian Theorem for

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Find A^{-1} and B, where

$B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

(b) Solve the system of given equations using LU decomposition method

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4$$

(c) Determine the solution for the given set of simultaneous linear equations using Gauss-Seidel Method

$$-3x + y = -2$$

$$2x - 3y + z = 0$$

$$2y - 3z = -1$$

(6+6+6)

4. (a) Use the ratio test to determine the convergence of following series with nth term as

$$a_n = \frac{n^2}{3^n}$$

(b) Find limit of following sequences and then deduce whether series is convergent, divergent or oscillatory

$$a_n = 1 + \frac{(-1)^n}{n}$$

(c) Use Cauchy's Integral test to determine the convergence of following series

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$$

(6+6+6)